1. For the equation $(x^2 - 5x + 6)\dfrac{dy}{dx} = y$,

   (a) what is the solution if the initial condition is $y(1) = 2$; (4 分)

   (b) what is the solution if the initial condition is $y(2) = 1$. (4 分)

2. (a) What is the meaning of the eq. $M(x,y)dx + N(x,y)dy = 0$ being said to be exact?   And what is the condition ? (4 分)

   (b) Find the general solution of the equation $\dfrac{dy}{dx} = \dfrac{(1 + y^2 + 3x^2 y)}{(1 - 2xy - x^3)}$. (8 分)

3. Consider the following inhomogeneous system

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 3, \\ 2x_1 + 3x_2 + 3x_3 = 4, \\ 3x_1 + x_2 - 2x_3 = 2, \end{cases}$$

   which can be written in the form $\vec{A}x = b$.

   (a) What is the rank of the coefficient matrix $\vec{A}$? (5分)

   (b) What is the rank of the augmented matrix $\vec{G}$? (2分)

   (c) Does the system have unique solution? Why? (3分)

   (d) Using Cramer's rule, find the solution. (7分)

4. Using Stoke's Theorem, find $\oint_{\gamma} \mathbf{v} \cdot dr$, where

   $\mathbf{v}(x,y,z) = (z - 2y)\mathbf{i} + (3x - 4y)\mathbf{j} + (z + 3y)\mathbf{k}$, and γ is the unit circle in the plane $z = 2$. (10 分)

5. Classify the following partial differential equations as hyperbolic, parabolic, or elliptic.

(a)
$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} + q(x,t)$$

(b)
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + f(x,t)$$

(8 分)

(c)
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = q(x,y)$$

(d)
$$\frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 u}{\partial x \partial y} + 9 \frac{\partial^2 u}{\partial y^2} = 0$$

6. Solve the Sturm-Liouvill prob.

$$u'' + \lambda u = 0, \qquad 0 < x < \pi$$

$$u(0) = 0, \qquad u'(\pi) = 0 \qquad (10 \text{ 分})$$

7. Solve the following initial-boundary value problem

EQ. : $u_t = \kappa u_{xx}$, $0 < x < l,$ $t > 0$

I. C : $u(x,0) = u_0$ (const.),

B. C's : $u(0,t) = 0,$ $u'(l,t) = 0.$ (10 分)

Remarks：總分 :75 分