-

104 年度第 1 次研究生入學能力考試試題

科目： 工程數學

考試日期： 104 年 4 月 11 日　　第 1 頁，共 2 頁

1. (a) Find the general solution of the equation $y' = y^2 - 4$ (5%)

   (b) What are the constant solutions for the above equation? Can you obtain them from the general solution? (10%)

2. (a) What is the meaning of the eq. $M(x,y)dx + N(x,y)dy = 0$ being said to be exact? And what is the condition？ (5%)

   (b) Find the general solution of the equation $\frac{dy}{dx} = \frac{(1 + y^2 + 3x^2 y)}{(1 - 2xy - x^3)}$. (10%)

3. (a) What is the meaning of the two functions $u_1(x)$, $u_2(x)$ being said to be linearly dependent (or independent) over a given interval. (5%)

   (b) By the above meaning, derive the condition for linearly dependent (or independent). (10%)

4. Find the solution of the differential equation $y'' + 16y = 0,$

   If the boundary conditions are : (i) $y(0) = 0, \quad y(\pi/2) = 0;$

   (ii) $y(0) = 0, \ y(\pi/8) = 0;$ (iii) $y(0) = 0, \ y(\pi/2) = 1.$ (10%)

5. Consider the following inhomogeneous system

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 3, \\ 2x_1 + 3x_2 + 3x_3 = 4, \\ 3x_1 + \ x_2 - 2x_3 = 2, \end{cases}$$

   which can be written in the form $\vec{\vec{A}}\underset{\sim}{x} = \underset{\sim}{b}.$

   (a) What is the rank of the coefficient matrix $\vec{\vec{A}}$? (5%)

(b) What is the rank of the augmented matrix $\tilde{\tilde{G}}$? (2%)

(c) Does the system have unique solution? Why? (3%)

(d) Using Cramer's rule, find the solution. (8%)

(e) For the homogeneous system $\tilde{\tilde{A}}\underset{\sim}{x} = 0,$ Does it have trivial solution or nontrivial solution? Why? (2%)

6. Solve the Sturm-Liouvill prob.

$$u'' + \lambda u = 0, \qquad 0 < x < \pi$$

$$u(0) = 0, \qquad u'(\pi) = 0 \qquad (10\%)$$

7. Solve the following initial-boundary value problem

EQ. : $u_t = \kappa u_{xx}$, $0 < x < l, \quad t > 0$

I. C : $u(x,0) = u_0$ (const.),

B. C's : $u(0,t) = 0, \qquad u'(l,t) = 0.$ (15%)